

## Group velocity and phase velocity

Lecture notes (Feb, 2015) : M. S. Santhanam  
IISER, Pune

*This notes is meant to be a supplement and not a substitute for standard text books.*

### 1 Phase and group velocity of waves

To understand the difference between phase and group velocity of waves, consider the following analogy. A group of people, say city marathon runners, start from the starting at the same time. Initially it would appear that all of them are running at the same speed. As time passes, group spreads out (disperses) simply because each runner in the group is running with different speed. If you think of phase velocity to be like the speed of an individual runner, then the group velocity is the speed of the entire group as a whole. Obviously and most often, individual runners can run faster than the group as a whole. To stretch this analogy, we note that the phase velocity  $v_p$  of waves are typically larger than the group velocity  $v_g$  of waves. However, this really depends on the properties of the medium. The media in which  $v_g = v_p$  is called the non-dispersive medium. But the media in which  $v_g < v_p$  is called normal dispersion. The media in which  $v_g > v_p$  is called anomolous dispersive media. It must be emphasised that dispersion is a property of the medium in which a wave travels. It is not the property of the waves themselves.

The relation between phase and group velocity is given by,

$$v_g = \frac{d\omega}{dk} = v_p - \lambda \frac{dv_p}{d\lambda}$$

Generally,  $\omega(k)$  is called the dispersion relation and indicates the dispersion properties of a medium. As this formula predicts, if the phase velocity does not depend on the wavelength of the propogating wave, then  $v_g = v_p$ . For example, sound waves are non-dispersive in air, i.e, all the individual components that make up the sound wave travel at same speed. Phase velocity of sound waves is independent of the wavelength when it propogates in air.

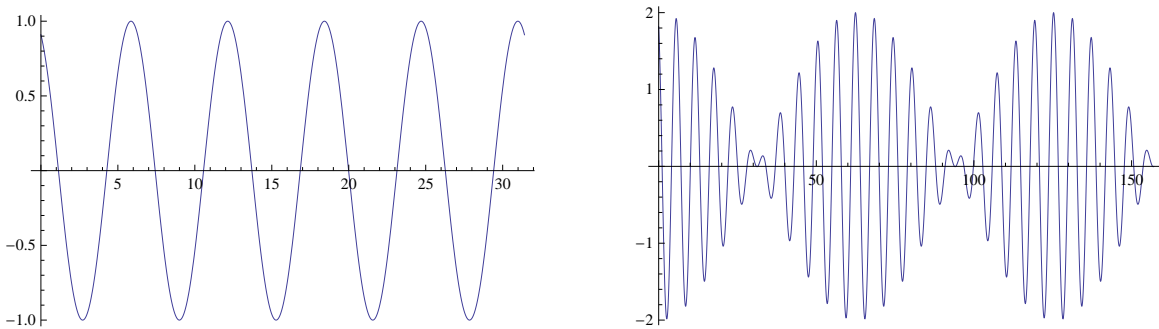


Figure 1: (left) A single travelling wave with frequency  $\omega = 1$ . (right) A group of waves composed of two waves with frequencies  $\omega = 1$  and  $\omega = 1.1$ .

Briefly, phase velocity refers to the velocity of a monochromatic wave, let's say, the velocity of one of the peaks of the wave. For example, a monochromatic wave with angular frequency  $\omega = 2\pi\nu$  ( $\nu$  is the frequency) travelling in +ve x-direction is given by,

$$y = A \sin(\omega t - kx).$$

On the other hand, group velocity refers to a *group* composed of waves within a frequency band  $\Delta\omega$ . Group velocity is the velocity with which the entire group of waves would travel.

The following figure 1 shows  $y = \sin(2 + t)$  and  $y = \sin(2 + t) + \sin(2 + 1.1t)$ . The last form is the sum of two waves whose frequencies differ by 0.1. Notice that the amplitude of the group is modulated as a function of  $t$ . The example here shows the waves as a function of  $t$ , but similar scenario holds good for waves as a function of  $x$ . For the travelling wave shown in the left panel of the figure, phase velocity is the velocity with which any one of the peaks progresses. However, for the right panel of the figure, the speed of any of the peaks would give the group velocity.

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